Reactor Physics of Pulsing: Fuchs - Hansen Adiabatic Model

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The purpose of this presentation is:

- physical description of pulse experiment
- derivation of Fuchs-Hansen adiabatic model
- comparison of the theoretical model with the experimental results.

Application:

- (TRIGA) research reactors: pulse mode operation, normal operation
- power reactors: reactivity transient, severe accident.

Detailed explanation is found in the following references:

1. Bell-Glasstone: Nuclear Reactor Theory, Chapter 9.6. Large power excursions

2. I.Mele, M. Ravnik, A. Trkov, “TRIGA Mark II Benchmark Experiment, Part II: Pulse Operation,” Nuclear Technology 105, 52-58, 1994.

3. M. Ravnik, Experimental Verification of Adiabatic Fuchs-Hansen Pulse Model, 4th Regional Meeting Nuclear Energy in Central Europe, September 7-10, 1997, Bled, Slovenia, 450-456.

4. M. Ravnik et al., PULSTRI-1, A computer program for TRIGA reactor pulse calculations, IJS-DP-5756, January 1990.

Physical description of pulsing in TRIGA reactor

Schematically:

Large reactivity insertion >>>

>>>prompt criticality >>>

>>>power increasing>>>

>>>fuel temperature increasing>>>

>>>reactivity decreasing due to negative temperature reactivity effect ('feedback' reactivity)>>>

>>>power decreasing, temperature increasing>>>

>>>further reactivity decreasing to zero or even negative value>>>

>>>power stabilizing at (relatively) low level.

Reactivity transient from $\rho = 0 \rightarrow \rho >> \beta$ in 0.1sec result of:

- fast withdrawal of transient rod by means of pneumatic system (pulse experiment)
- rod ejection in PWR (hypothetical transient in power reactor)

Consequences:

- $\rho >> \beta$, reactor prompt critical $\rightarrow$ exponential power increase with short period (10-2 s)
- Released thermal energy accumulates in fuel:

  heat generation >> conductive heat removal
- fast fuel temperature increasing
- fast internal gas pressure increasing
- fast thermal expansion of fuel meat (but not the cladding)

leading to fuel damage and disintegration if energy released in pulse exceeds design limit.

TRIGA reactors designed for pulsing. Pulse energy in TRIGA reactors limited by

negative and prompt fuel temperature reactivity effect

Negative: due to spectrum hardening and Doppler effect

typical value $\alpha_f \equiv \frac{d\rho}{dT} = -0.01 \ \beta/°C = -7 pcm/°C$

Prompt: due to homogeneous reactor

uranium homogeneously mixed with hydrogen,

fission and moderation in fuel

Practical pulse procedure

- reactor critical with transient rod completely inserted,
- low power (100W)
- transient rod upper position preset by adjusting vertical position of the piston stopper
- other rods withdrawn (except for the part compensating excess reactivity)
- preset SCRAM approx. 5sec after pulse
- start pulse (Fire signal)
    - valve from air high pressure tank opens
    - air pressure moves transient rod drive mechanism piston
    - transient rod connected to the piston moves out until upper stopper is reached (typically in 0.1sec)

- pulse (from 0.1sec to 1sec after start signal, depending on inserted reactivity)
- SCRAM after preset time (typically 5sec)
- cooling (typically 15minutes)
- repeat procedure

Inserted reactivity regulated by transient rod upper position - defined by the operator

'Feedback' reactivity depends on fuel temperature and temperature reactivity coefficient:

$$feedback\ reactivity \approx \alpha_f * \Delta T \equiv \frac{\partial \rho}{\partial T} * \Delta T$$

If Δ T small (small pulse) , then feedback reactivity smaller than inserted reactivity

>>>> reactor remains critical at power defined by the power defect (reason for SCRAM)

If Δ T large (large pulse), then feedback reactivity greater than inserted reactivity

>>>> reactor becomes subcritical after the pulse, but would return to power after short time due to cooling (reason for SCRAM)

Time dependence of reactivity, power and energy are presented in Fig. 1 and Fig. 2

Physical model of the pulse -
Fuchs - Hansen adiabatic model

Assumption 1: **point kinetics approximation**

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{l} P(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{l} P(t) - \lambda_i C_i(t), \ldots\ldots i = 1,6$$

P total reactor power (MW)

λ_i delayed neutron precursors' decay constant (in average approx. $0.1s^{-1}$, $1/\lambda_i$= 10s)

l prompt neutron generation time (40μ sec << $1/\lambda$ i)

β effective delayed neutron fraction (importance factor times nuclear delayed neutron fraction for U-235, in TRIGA: from 0.0070 to 0.0073)

ρ reactivity, convenient unit: β = \$ = 0.007= 700pcm

Assumption 2: **Contribution of delayed neutrons during pulse negligible**

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{l} P(t)$$

Valid only if $\rho >> \beta$

Assumption 3: **Transient rod withdrawal time is short, rod is withdrawn before temperature feedback effect on reactivity**

For convenience we introduce

$\rho' = \rho(t=0) - \beta$ as pulse prompt reactivity.

Assumption 4: **Reactivity decrease during pulse is proportional to accumulated energy**

$$\rho(t) - \beta = \rho' - E(t) = \rho' - \gamma \int_0^t P(t')dt'$$

Assumption implies that

- there is no energy transfer from multiplying part of the fuel element during pulse (adiabatic approximation) ?
- coefficient γ constant, independent of temperature

Evidently

$$\gamma = -\frac{\alpha_f}{mc_p}$$

α_f is fuel temperature reactivity coefficient,

c_p specific thermal capacity of fuel (multiplying material only)

m total mass of fuel (multiplying material) in reactor.

It is assumed that α_f and c_p do not depend on temperature. Inserting ρ (t) into equation for P we obtain Fuchs-Hansen model

$$\frac{dP(t)}{dt} = P(t)\left[\alpha_0 - b\int_0^t P(t')dt'\right]$$

$$\alpha_0 = \frac{\rho'}{l}$$

$$b = \frac{\gamma'}{l}$$

Initial condition P(0)=Po

For t=0 we see immediately:

$$\frac{dP(t)}{dt} = \alpha_0 P(t) \Rightarrow P(t) = P_0 e^{\alpha_0 t}$$

>>>> exponential power increasing.
αo is initial inverse period, 0.007/40μ s = 175 sec^{-1} = 1/ (0.005sec)

Formal solution of FH equation

Kinetics equation

$$\frac{dP(t)}{dt} = P(t)\left[\alpha_0 - b\int_0^t P(t')dt'\right]$$

can be solved analytically by introducing a new variable y(t)

$$\left[\alpha_0 - b\int_0^t P(t')dt'\right] = y(t)$$

.

If we make second derivative of y(t) we obtain

$$\frac{dy(t)}{dt} = -bP(t) \Rightarrow P(t) = -\frac{1}{b}\frac{dy(t)}{dt}$$

$$\frac{d^2y(t)}{dt^2} = -b\frac{dP(t)}{dt} \Rightarrow \frac{dP(t)}{dt} = -\frac{1}{b}\frac{d^2y(t)}{dt^2}$$

Inserting P(t) and dP/dt into initial equation yields nonlinear second order differential equation

$$\frac{d^2y}{dt^2} = y\frac{dy}{dt}$$.

It can be integrated

$$\frac{dy}{dt} = \frac{1}{2}y^2 + const. \quad = \frac{1}{2}\left(y^2 - c^2\right)$$

Constant c can be determined from the initial condition at t=0

$$\frac{dy}{dt}\Big|_{t=0} = -bP(0) = -bP_0$$

$$y(0) = \alpha_0 \qquad ?$$

$$-bP_0 = \frac{1}{2}(\alpha_0^2 - c^2) \Rightarrow c = \sqrt{\alpha_0^2 + 2bP_0}$$

(negative solution for c is omitted as it has no physical meaning)

Equation

$$\frac{dy}{dt} = \frac{1}{2}\left(y^2 - c^2\right)$$

is solved by introducing a new variable u(t)

$$y(t) = \frac{1}{u(t)} + c$$.

After insertion we obtain

$$-\frac{1}{u^2}\frac{du}{dt} = \frac{1}{2}\left(\frac{1}{u^2} + \frac{2c}{u} + c^2 - c^2\right) = \frac{1}{2}\left(\frac{1}{u^2} + \frac{2c}{u}\right)$$

Multiplying with u^2 (u not 0) yields a simple non-homogeneous first order differential equation

$$\frac{du}{dt} + cu = -\frac{1}{2}$$.

Its solution:

$$u(t) = -\frac{1}{2c} + a \cdot e^{-ct}$$

Free coefficient a is calculated from the initial condition at t=0

$$u(o) = \frac{1}{y(0) - c} = \frac{1}{\alpha_0 - c} = -\frac{1}{2c} + a \Rightarrow$$

$$a = -\frac{1}{2c} \cdot \frac{c + \alpha_0}{c - \alpha_0} \equiv -\frac{1}{2c} \cdot A$$

By reinserting u and y into P(t) we get final solution:

$$P(t) = \frac{2c^2 A e^{-at}}{b\left(Ae^{-at} + 1\right)^2}$$

$$E(t) = \frac{\alpha_0 + c}{b}\left[\frac{1 - e^{-at}}{Ae^{-at} + 1}\right]$$

where

$$c \equiv \sqrt{\alpha_0^2 + 2bP_0}$$

$$A \equiv \frac{c + \alpha_0}{c - \alpha_0}$$

Total released energy E_t:

$$E_t = E(t \to \infty) = \frac{2\alpha_0}{b} = \frac{2\rho'}{\gamma}$$

Maximum power in pulse P_{max}:

$$P_{max} = \frac{(\rho')^2}{2l\gamma}$$

Note: E_t and P_{max} are zero for inserted reactivity up to 1$ - the model is obviously not valid for reactivity insertions smaller than prompt reactivity

According to Fuchs-Hansen model

$P_{max} \propto \rho$ '2

- inversely proportional to l
- inversely proportional to γ
- inversely proportional to α_f
- proportional to m (mass of reactor fuel)

Et $\propto \rho$ '

inversely proportional to γ

inversely proportional to α_f

proportional to m (mass of reactor fuel)

Comparison of model and experiment

Comparison of measured and theoretical results is presented in Figures 3 - 6

Discrepancies between measured and calculated results are consequence of deficiencies

in the physical model

in the measurement.

Main deficiencies of the physical model:

- temperature reactivity coefficient and fuel thermal capacity independent of temperature

- delayed neutrons (and their power) neglected, important at low reactivity insertions and for energy released after pulse

- point kinetics (more important in big reactors)

Source of systematic experimental errors:

- pulse channel calibration and sensitivity of the detector on the local flux variations

- transient rod reactivity worth and influence of other control rods

- fuel burn-up due to long term steady state operation between pulsing, xenon effect

- modifications in core configuration.

Conclusion

Fuchs-Hansen adiabatic model and pulse experiment provides good insight in reactor physics of reactivity transient important in power reactor safety analysis.

Figures

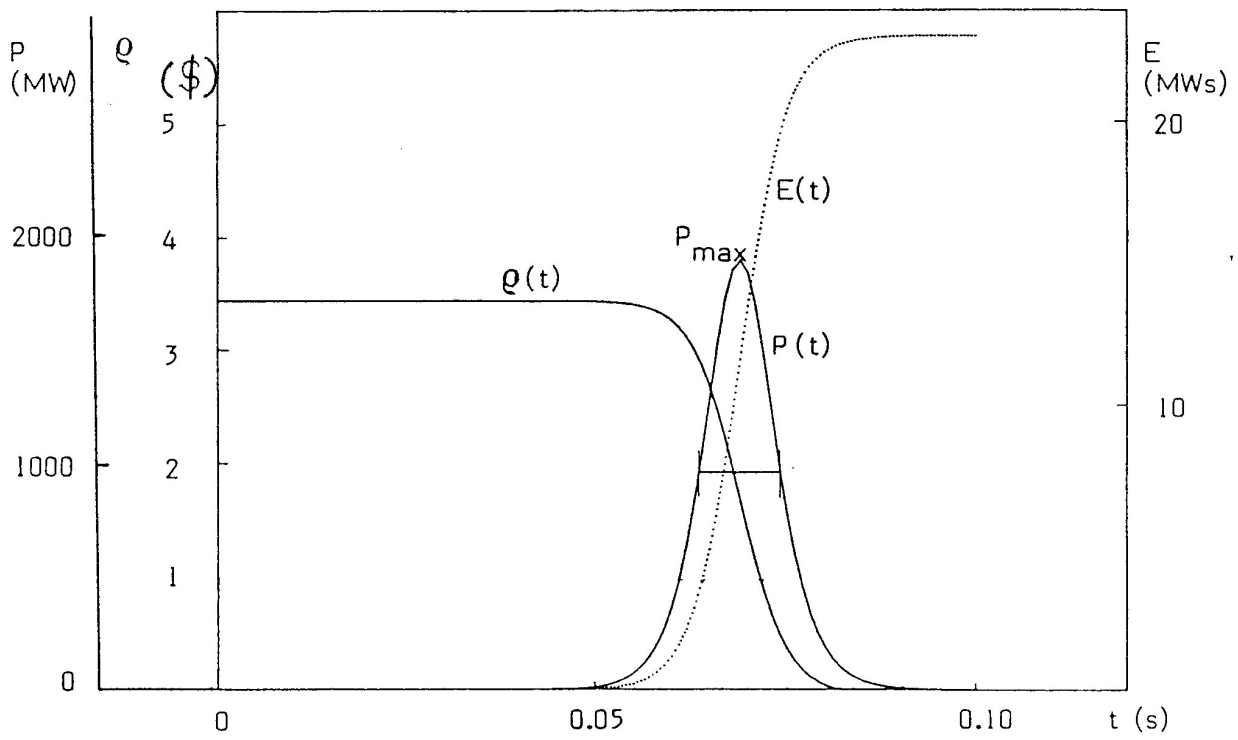

Figure 1. Dependence of reactivity, power and energy on time during pulse (schematically)

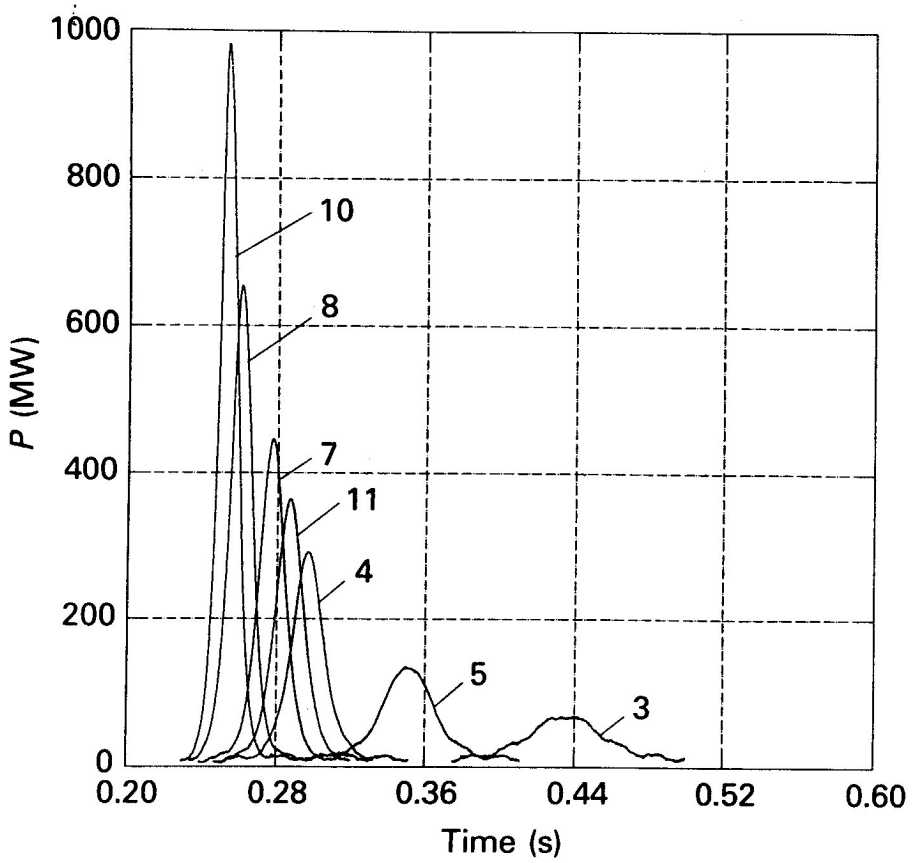

Figure 2. Measured P(t) for several pulses with different inserted reactivity

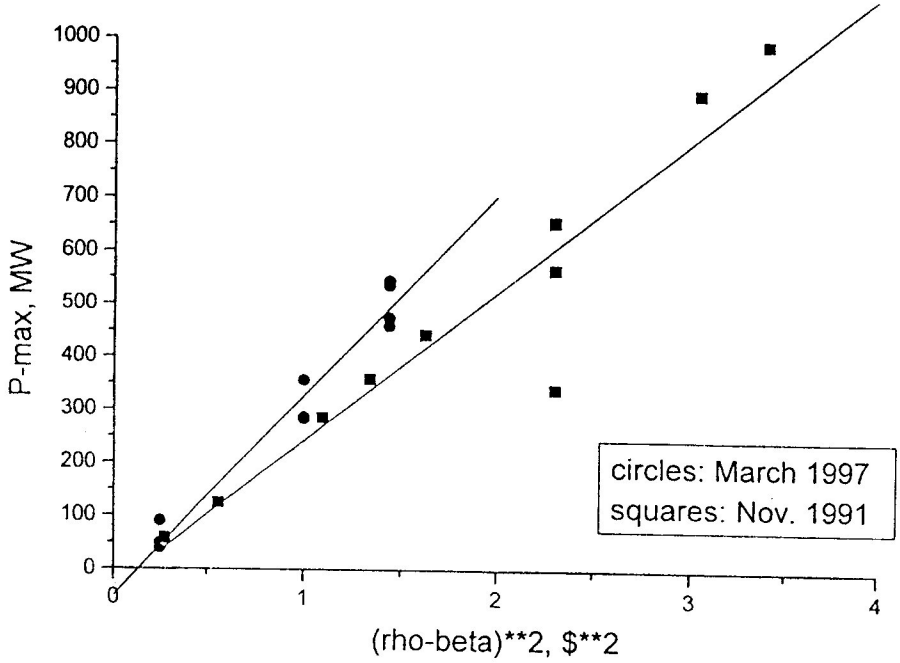

Figure 3. Measured and calculated maximum power P_{max} in dependence of ρ'^2

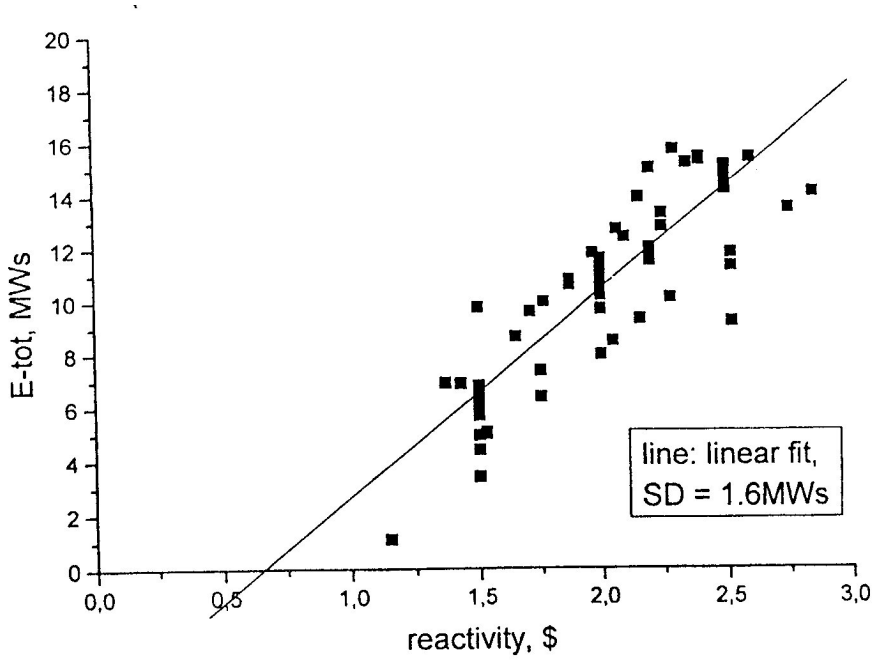

Figure 4. Measured and calculated total energy E_t in dependence of ρ'

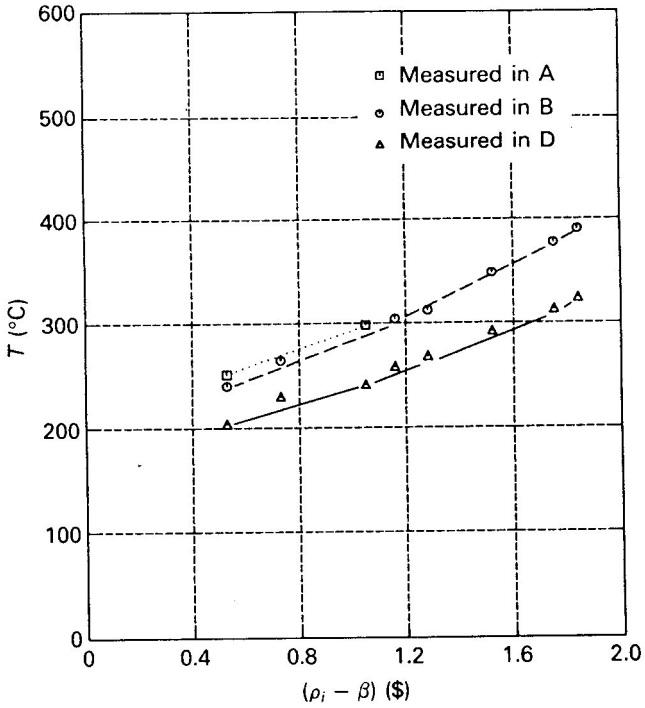

Figure 5. Measured maximum fuel temperature in dependence of ρ '

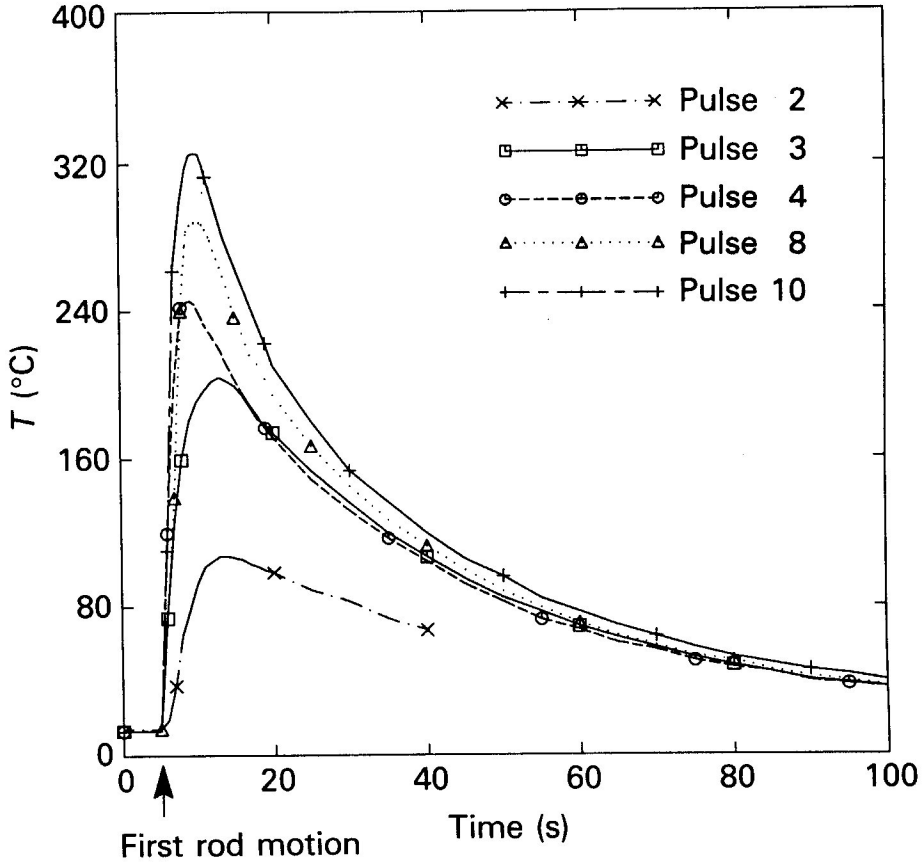

Figure 6. Measured fuel temperature in dependence of time after pulse